ME 3260 – Wind Machine Project

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Nomenclature

A = Swept Area of Blades (m^2)
B = Number of Blades
R = Total Blade Length (m)
C_L = Coefficient of Lift
C_{opt} = Optimal Chord Length (m)
P_{wind} = Wind Power (W)
r = Distance Along Blade Length (m)
u = Wind Speed (m/s)
u_r = Wind Speed at Location r (m/s)
z = Elevation (m)
z_r = Elevation at Location r (m)
ρ = Air Density (kg/m^3)
α = Optimal Angle of Attack (°)
θ = Angle of Twist (°)
ϕ = Relative Wind Speed angle(°)
λ = Tip Speed Ratio
λ_r = Relative Tip Speed (m/s)

1 Objective & Introduction

This report will detail the process of researching and calculating the most efficient parameters of a wind turbine blade for a wind turbine in a given environment. The constraints of the wind turbine are a given wind profile and maximum rotor height. The parameters that are being designed are blade length, ideal angle of attack, and the chord length and twist as a function of blade length. In this report, an overview of previous work will be done, which details the reasoning and sources of the assumptions used to determine the wind blade design. Next, the report will use those assumptions and resources to define the equations that were used to calculate the wind blade geometry, along with the calculations themselves. Finally, the report will discuss the final wind blade design and summarize the process that was conducted to arrive at the final parameters.

2 Overview of Previous Work

A few assumptions and simplifications were made in the process of designing the wind turbine blade.

The first assumption that was made was that the rotor would be at the maximum height of 50 meters. Since wind power is a function of wind speed and wind speed is a function that increases with altitude, it makes sense that the optimal power generating wind turbine design would have a rotor placed at the maximum height.

On average, the length of a wind turbine blade is approximately half the rotor height, the chosen length of the wind turbine blade for this design will be 25 meters.[1]

Although the turbine encompasses only half the available area a rotor height of 50 meters has to offer, many issues of having a blade too close the ground were avoided. Some examples are: turbulent drag-inducing wake effects and variable wind patterns near the ground which can create non-uniform loading on the blades and increasing wear and tear on components. Collision hazards from people or objects coming in contact with the low-lying blades. Ice shedding in colder climates. And greater deflection in the flex of the blade if not manufactured with enough support, which is more expensive.[2]

Another assumption was that the wind turbine blade airfoil shape would follow the design shown in Figure 1. This airfoil shape was designed for wind turbine blades up to 25 meters long, and is therefore applicable to the blade design in this report [3]. The next assumption made is to only use one airfoil profile for the entire blade, the NREL S818 airfoil. This was determined by the fact that it encompasses the largest length section of the blade out of the three airfoil shapes in Figure 1.

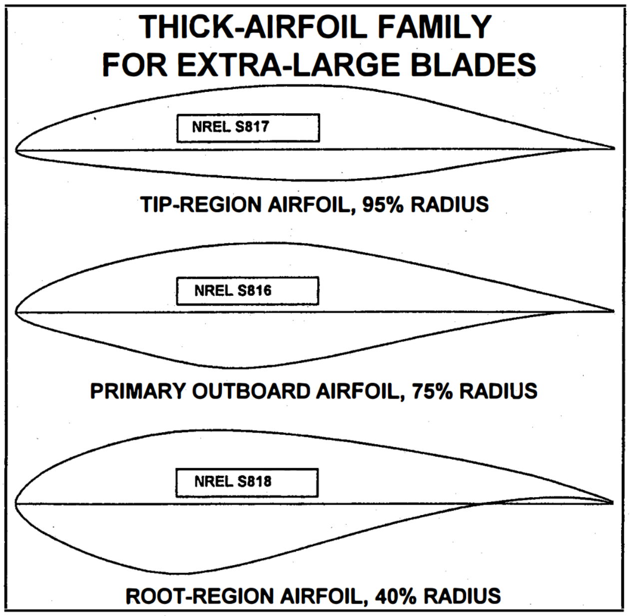


Figure 1: NREL S817, S816, and S818 airfoil shapes [3].

A graph of tip-speed ratio vs coefficient of performance, which was shown at the 11th International Conference on Future Environment and Energy and depicted in Figure 2, was used to determine that the combination of three turbine blades and a tip-speed ratio of 8 would result in the highest coefficient of performance.[4]

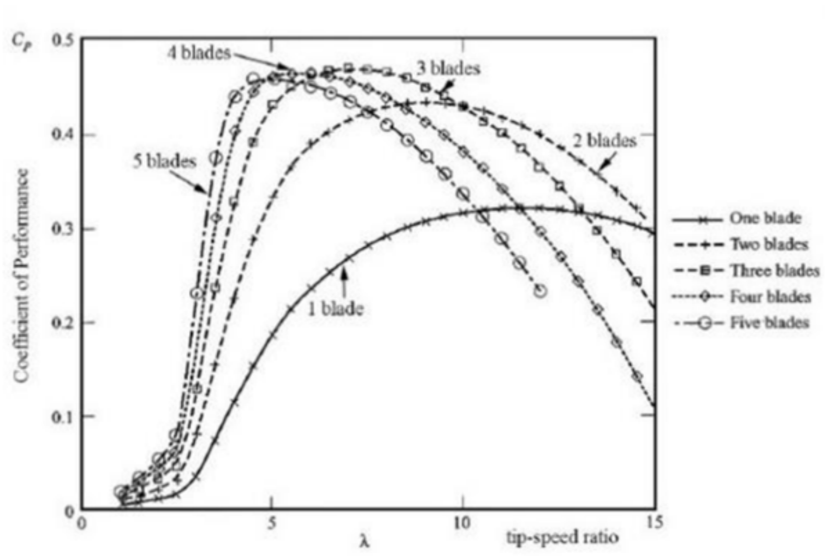


Figure 2: Coefficient of performance for different numbers of blades as a function of tip-speed ratio.[4]

To calculate the chord length and relative wind angle for the wind turbine blade along the length of the blade, the Duran Manwell model gives Equation 1, which was identified as one of the most effective formulas to apply. This due to the Duran Manwell model being ideal for power output when factors such as blade weight, design modification, and near root manufacturing can be ignored. [5]

$$C_{opt} = \frac{8\pi r}{BC_L}(1 - cos\phi), \phi = \frac{2}{3}\tan^{-1}\left(\frac{2}{3\lambda_r}\right) \tag{1}$$

3 Calculations

The calculations performed in the design process were completed and displayed using Python. The code is attached to the end of this report.

3.1 Wind Speed & Power

Equation 2 was given as the wind speed profile, where z is the height, u_r is 8 m/s, and z_r is 10 meters. The wind speed at a given height is shown in Figure 3.

$$\frac{u}{u_r} = \left(\frac{z}{z_r}\right)^2 \tag{2}$$

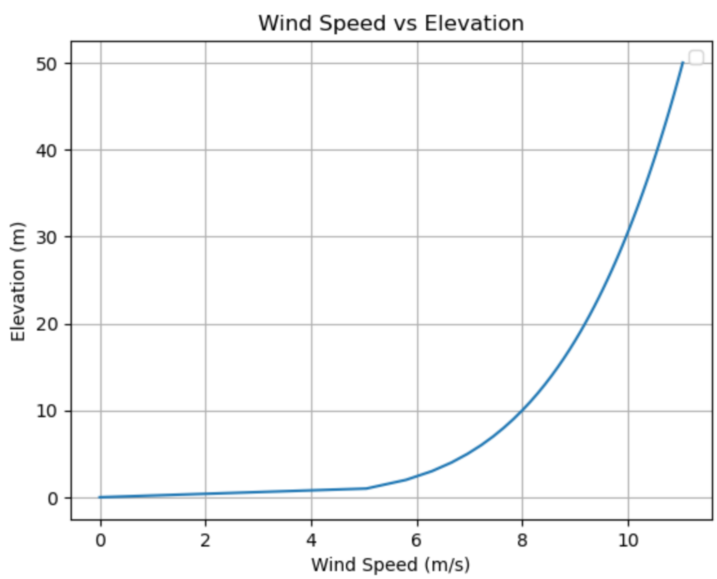


Figure 3: Wind speed vs elevation

Equation 3 was used to calculate the power of the wind at a given height, with u as the wind speed as a function of height, ρ as the density of the air, and the area A being a circle with the radius of the blade length, 25 meters. The power of the wind at a given height is shown in Figure 4.

$$P_{wind} = \frac{1}{2}\rho A u^3 \tag{3}$$

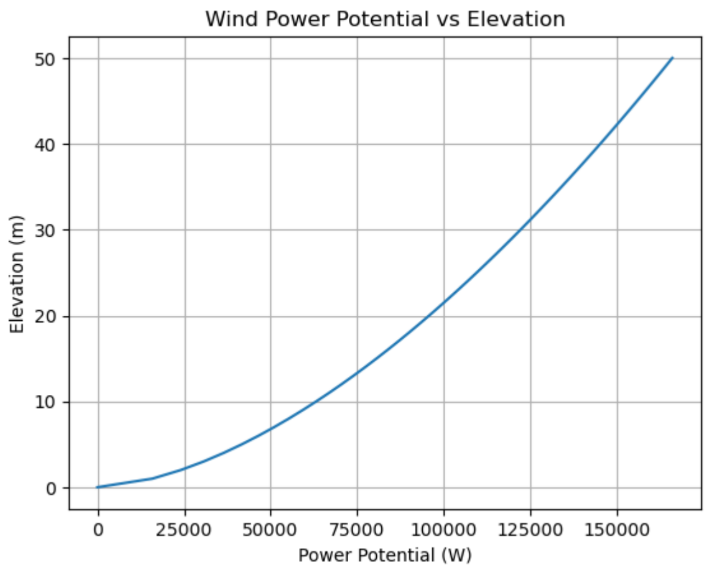


Figure 4: Wind power vs elevation

3.2 Ideal Angle of Attack

To find the ideal angle of attack of the wind turbine blade, tabulated data on the chosen air foil was analyzed and maximized.

Airfoil data on the NREL S818 was pulled from Airfoiltools.com and used to find a ratio of $\frac{C_L}{C_D}$ over a range of angles of attack. From this list of lift/drag ratios, the maximum value corresponds to the otimal angle of attack. The opimal angle of attack was found to be 9.45 degrees.[6]

3.3 Angle of Twist

To calculate the angle of twist and chord length, the specific tip speed is required. This can be calculated in Equation 4, where λ is the tip speed ratio, r is the distance along the blade length, and R is the total blade length.

$$\lambda_r = \lambda \left(\frac{r}{R}\right) \tag{4}$$

The angle of twist is then calculated using Equation 5, where ϕ is the relative wind speed angle and α is the optimal angle of attack. ϕ can be found using Equation 6.

$$\theta = \phi - \alpha \tag{5}$$

$$\phi = \frac{2}{3}\arctan\left(\frac{2}{3\lambda_r}\right) \tag{6}$$

This calculation (Equation 5) was made at 50 sections along the length of the blade. Then, the angle of twist as a function of distance from the rotor was plotted, as shown in in Figure 5.

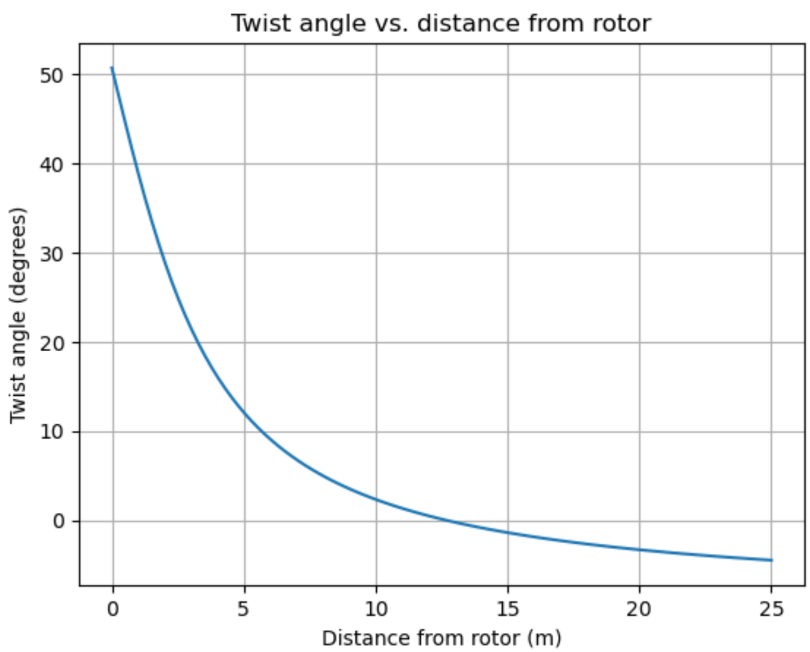


Figure 5: Angle of twist vs. distance from rotor

3.4 Chord Length

Equation 7 was used to determine the optimal chord length at a given radius, where C_{opt} is the optimal chord length, r is the distance along the blade length, B is the number of blades, C_L is the coefficient of lift at the optimal angle of attack, ϕ is the relative wind speed angle, λ_r is the relative tip speed, and R is the total blade length.

$$C_{opt} = \frac{8\pi r}{BC_L}(1 - cos\phi) \tag{7}$$

Figures 6 and 7 show the chord length as a function of their distance from the rotor, starting from zero and centered around zero respectively.

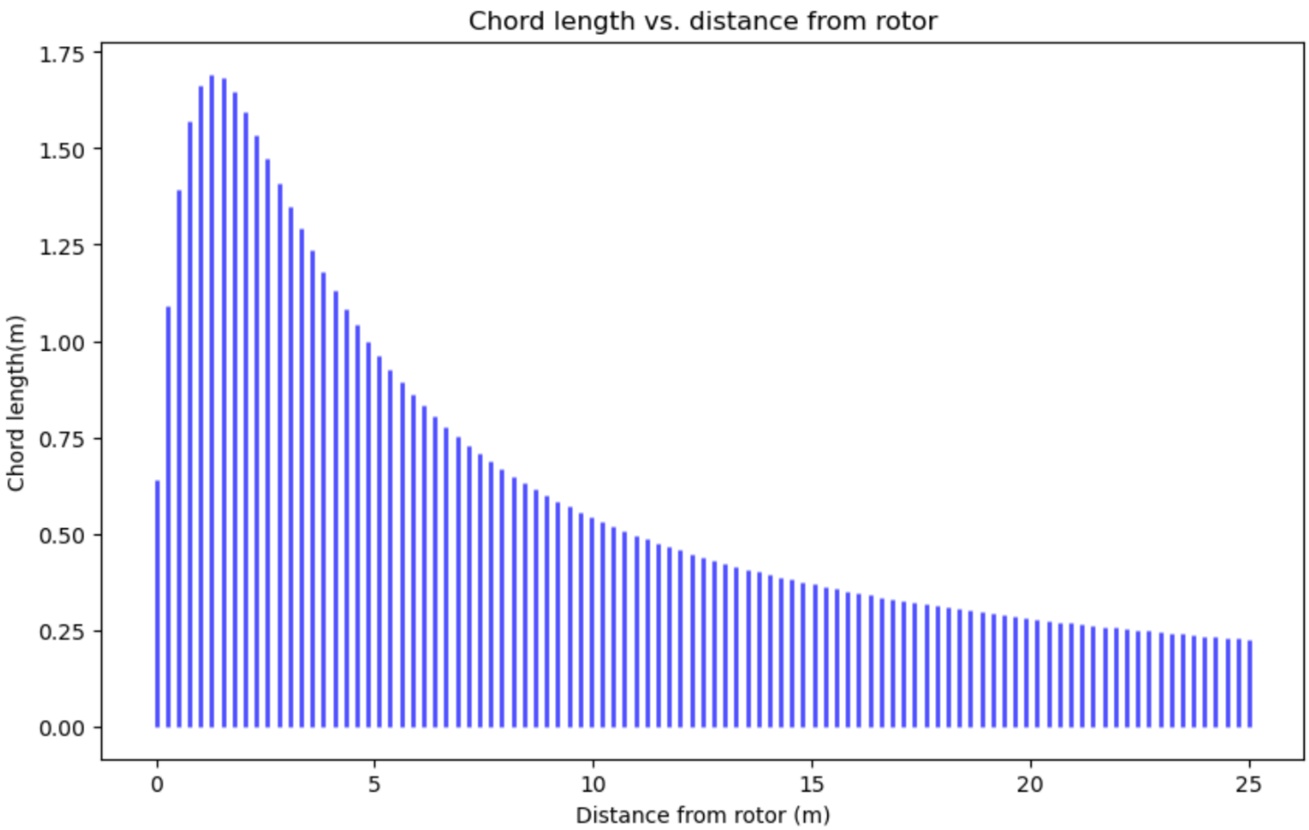


Figure 6: Chord length vs. distance from rotor, from 0.

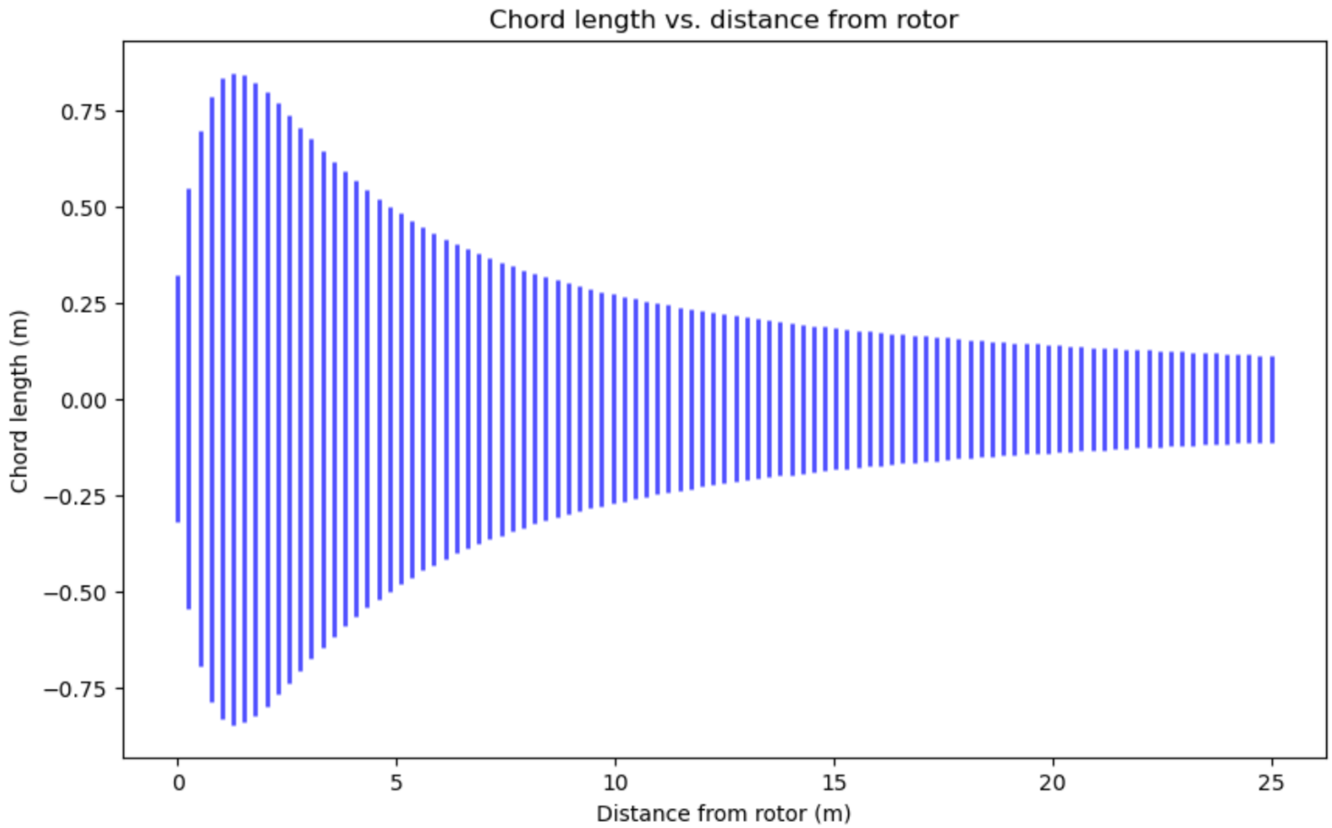


Figure 7: Chord length vs. distance from rotor, centered at 0.

Finally, the rotor was plotted in 3D space, with each of the axes representing physical space, shown in Figure 8. This was to display the twist angle of the blade as the length increases.

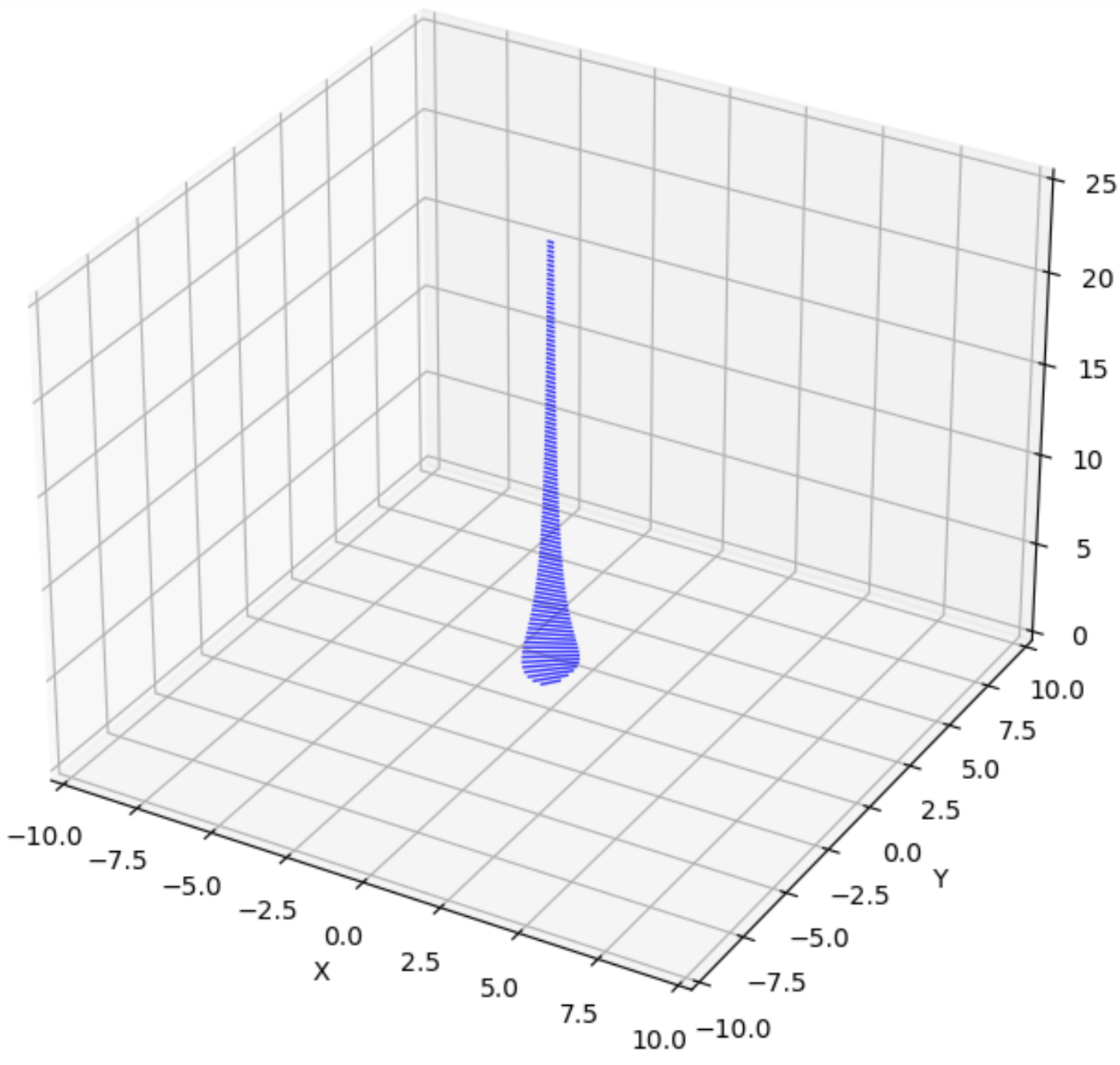


Figure 8: 3D model of blade

4 Results and Conclusions

The design and calculations of the three wind turbine blades have been completed based on the assumptions and methods outlined in the previous sections. Numerical results were obtained for optimal blade length, chord length, angle of attack, and angle of twist, determined using the Duran Manwell model and validated through graphical representations using Python.

4.1 Results

The optimal blade length was determined to be 25 meters long [1], extending from the rotor, which was based on the maximum rotor height constraint and standard blade-to-height ratios. An optimal angle of attack was selected to be 9.45°. This was decided by comparing attack angles which would maximize the lift to drag ratio of the NREL S818 airfoil shape. The greater the coefficient of lift in comparison to drag, the faster the blades will be propelled from the incoming wind and spin the generator shaft to produce the most power. The wind speed and power were calculated as functions of elevation from the base of the wind turbine with Equations 2 and 3 respectively. The exponential relationship for each characteristic is shown in Figures 3 and 4 respectively. The twist angle and chord length were calculated using Equations 5 and 7 respectively, as functions of blade length. The results of these calculations were compared by plotting their magnitude with their location from the rotor, seen in Figure 5 and 6 respectively. Utilizing python to plot all characteristics of the blade together for visual validation, Figure 8 shows a 3D representation of the designed turbine blade.

Under the given constraints, the Duran Manwell method proved to be a reliable framework for finding the optimized angle of attack and chord length which presented the highest turbine efficiency. The parameters calculated align with present day design standards and models well with characteristics of turbines which have similar dimensions. Based on Figure 2, the selected tip-speed ratio of 8 combined with three blades, yielded the highest coefficient of performance. Wind speed profiles calculated indicate that positioning the rotor at the maximum height constrained (50 meters) benefits the most from the higher wind speeds, which significantly enhances power output of the turbine. Taking into account what is well-suited for this application including all power maximizing properties of the turbine itself, the NREL S818 airfoil profile which could capitalize on all three and streamline manufacturing was retained.

4.2 Conclusions

Given environmental and structural constraints, the design process for analytical and visually validated results successfully demonstrated optimization between power generation, manufacturing complexity cost and risks which affect the performance, safety, and longevity of the turbine. The combination of a 25 meter blade length, 9.45° angle of attack, and altering specific chord length and twist profile to maintain an optimal angle of attack along the entire length, developed a final design which maximizes the power generation potential while maintaining structural feasibility. While building blade geometry, the use of computational tools and graphical analysis facilitated accurate and efficient design iterations. This report's justification of assumptions and reasonings build upon previous work cited but still fortify a foundation for future wind turbine designs. Expanding the model to include dynamic effects such as variable wind speeds and load variations in the future would pose greater accuracy in power generation calculations and potentially competitive alternate airfoil designs.

4.3 Extracurricular Design Adoptions

Some improvements to the design to address environmental concerns not elaborated on in this report are included below.

- Scientists have found that painting one blade of a turbine black, which can increase visibility, can reduce bird fatalities by more than 70 percent.[7]
- Low-noise bionic wind turbine blade based on owl wing and feather characteristic structure by comprising a non-smooth front edge structure and a curve sawtooth tail edge structure of the blade.[8]
- Designing a turbine which effectively reaches the maximum possible wind altitude and utilizes the maximum given wind power, up to Betz's limit, will establish an effective use of land which the device occupies.

References

[1] R. Wass, "Design of Wind Turbine Tower Height and Blade Length: an Design of Wind Turbine Tower Height and Blade Length: an Optimization Approach Optimization Approach," Sep. 2018. Available:
https://scholarworks.uark.edu/cgi/viewcontent.cgi?article=1070&context=meeguht

[2] Željko urišić and J. Mikulović, "Assessment of the wind energy resource in the South Banat region, Serbia," Renewable and Sustainable Energy Reviews, vol. 16, no. 5, pp. 3014–3023, Mar. 2012, doi: https://doi.org/10.1016/j.rser.2012.02.026.

[3] J. Tangier and D. Somers, "NREL Airfoil Families for HAWTs" 1995. Available: https://www.nrel.gov/docs/legosti/old/7109.pdf

[4] K. A. Adeyeye, N. Ijumba, and J. Colton, "The Effect of the Number of Blades on the Efficiency of A Wind Turbine," IOP Conference Series: Earth and Environmental Science, vol. 801, no. 1, p. 012020, Jun. 2021, doi: https://doi.org/10.1088/1755-1315/801/1/012020.

[5] Temesgen Batu and H. G. Lemu, "Comparative Study of the Effect of Chord Length Computation Methods in Design of Wind Turbine Blade," Lecture notes in electrical engineering, pp. 106–115, Jan. 2020, doi: https://doi.org/10.1007/978-981-15-2341-0_14.

[6] "NREL's S818 Airfoil (s818-nr)," airfoiltools.com. http://airfoiltools.com/airfoil/details?airfoil=s818-nr

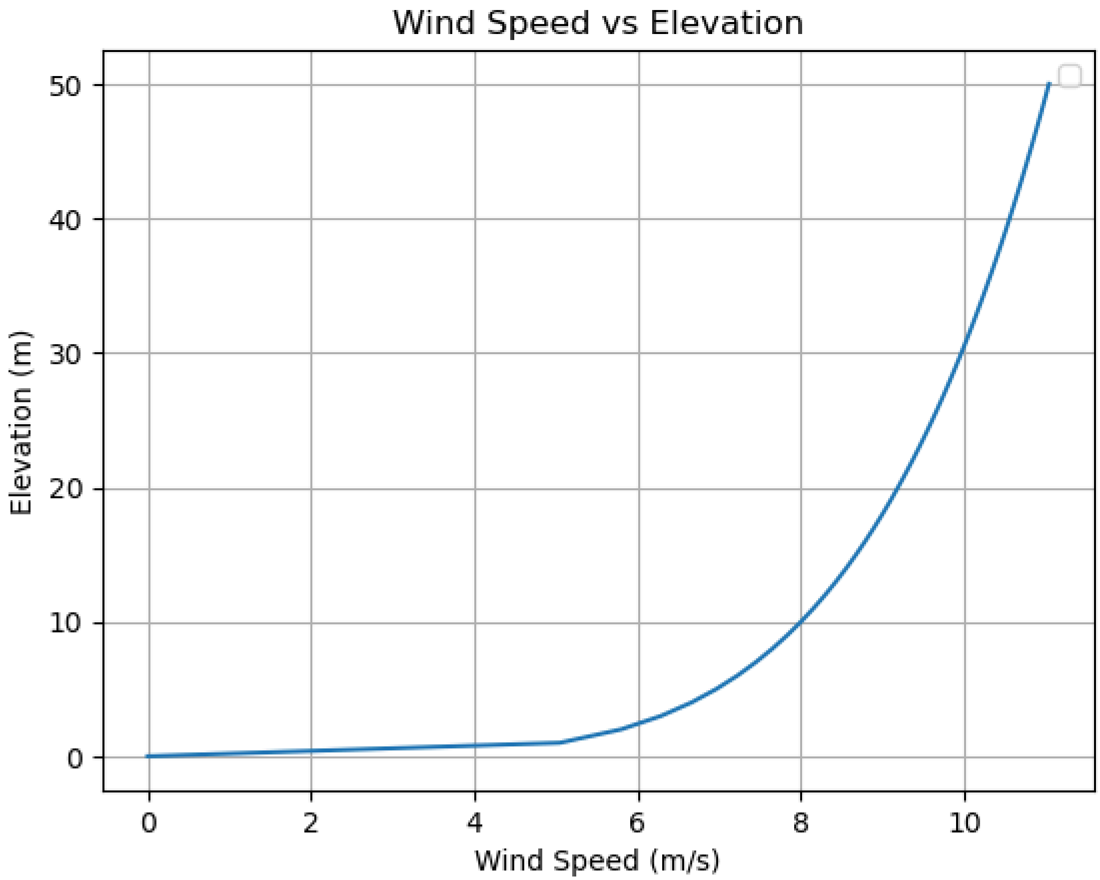
[7] "Do wind turbines kill birds?," MIT Climate Portal, 2023. https://climate.mit.edu/ask-mit/do-wind-turbines-kill-birds#: :text=There%20are%20also%20ways%20to,1 (accessed Dec. 10, 2024).

[8] "CN118128688A - Low-noise bionic wind turbine blade based on owl wing and feather characteristic structure - Google Patents," Google.com, Dec. 03, 2022.
https://patents.google.com/patent/CN118128688A/en (accessed Dec. 10, 2024).

Plot of wind speed vs height code

In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd

ur = 8
zr = 10
zrotor = 50

def wind_speed(z):
    return ur * ((z/zr)**0.2)

zrange = np.arange(0, zrotor+1)
u = []

for z in zrange:
    u.append(wind_speed(z))
plt.plot(u, zrange)
plt.xlabel("Wind Speed (m/s)")
plt.ylabel("Elevation (m)")
plt.title("Wind Speed vs Elevation")
plt.grid()
plt.legend()
plt.show()
plt.savefig("wind_speed_vs_elevation.png", dpi=300, bbox_inches='tight')
```

```
No artists with labels found to put in legend.  Note that artists whose label start with an underscore are ignored when legend() is called with no argument.
```

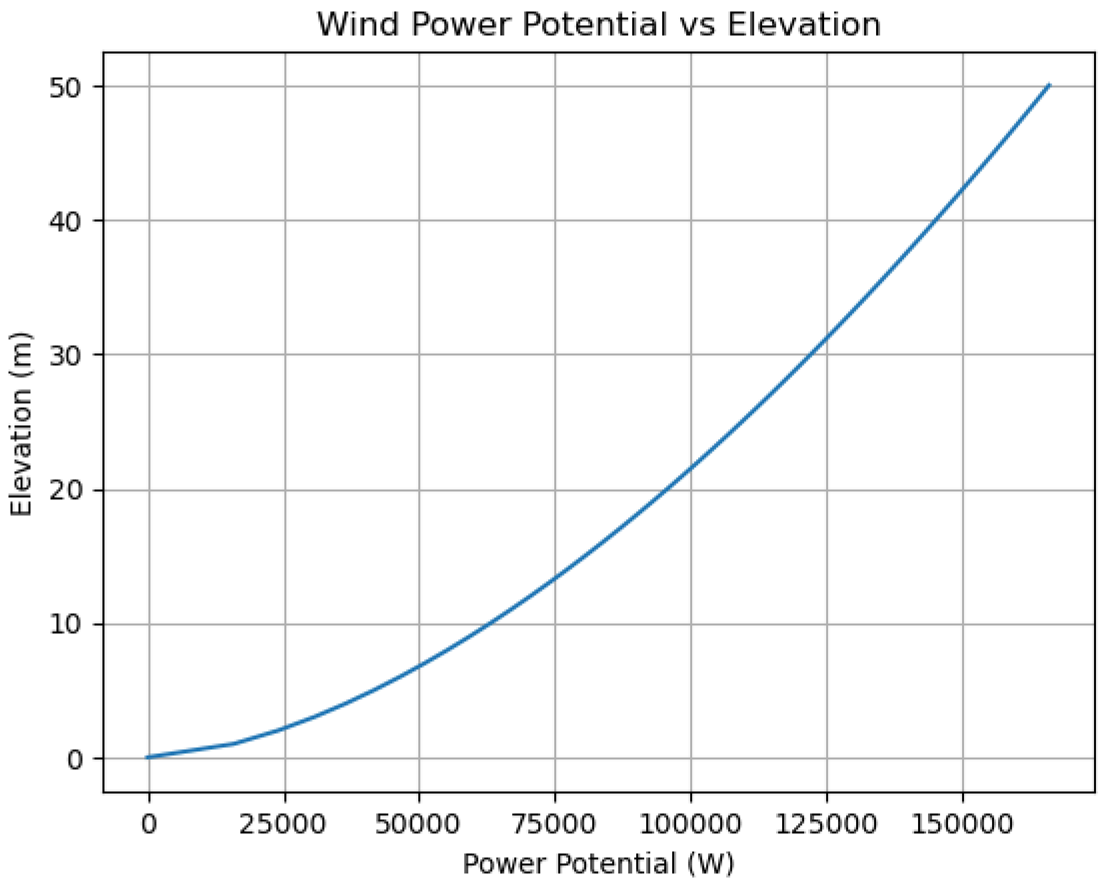



```
<Figure size 640x480 with 0 Axes>
```

Plot of wind power vs height code

In [2]:
```
rho = 1
R = 25
A = R * np.pi**2

windpowers = []
for z in zrange:
    windpowers.append(0.5 * rho * A * u[z]**3)
plt.plot(windpowers, zrange)
plt.ylabel("Elevation (m)")
plt.xlabel("Power Potential (W)")
plt.title("Wind Power Potential vs Elevation")
```

```
plt.grid()
plt.show()
```



Chord length calculation

This part reads the text file containing the C_L with respect to the α for our chosen air foil.

In [3]:
```
import pandas as pd


file_path = '/Users/samtull/Downloads/xf-s818-nr-1000000.txt'

#Reads file skipping past all the rows
df = pd.read_csv(file_path, skiprows=10, delim_whitespace=True)
```

```
#makes an array containing all alpha and CL values (skips the first line because it is a header)
alphas = df['alpha'].to_numpy()[53:].astype(float)
CLs = df['CL'].to_numpy()[53:].astype(float)
CDs = df['CD'].to_numpy()[53:].astype(float)
optaoa = alphas[np.argmax(CLs/CDs)]
CL = CLs[np.argmax(CLs/CDs)]




pd.set_option('display.max_rows', None)  # Show all rows
pd.set_option('display.max_columns', None)  # Show all columns
```

Duran Manwell Model for calculating chord length

$$C_opt = \frac{8\pi r}{BC_L}(1 - cos\phi), \phi = \tan^{-1}\left(\frac{2}{3\lambda_r}\right)$$

C_opt = Optimal chord length

r = distance along blade length

B = Number of blades

C_L = Coefficient of lift at optimal angle of attack

ϕ = Relative wind speed direction

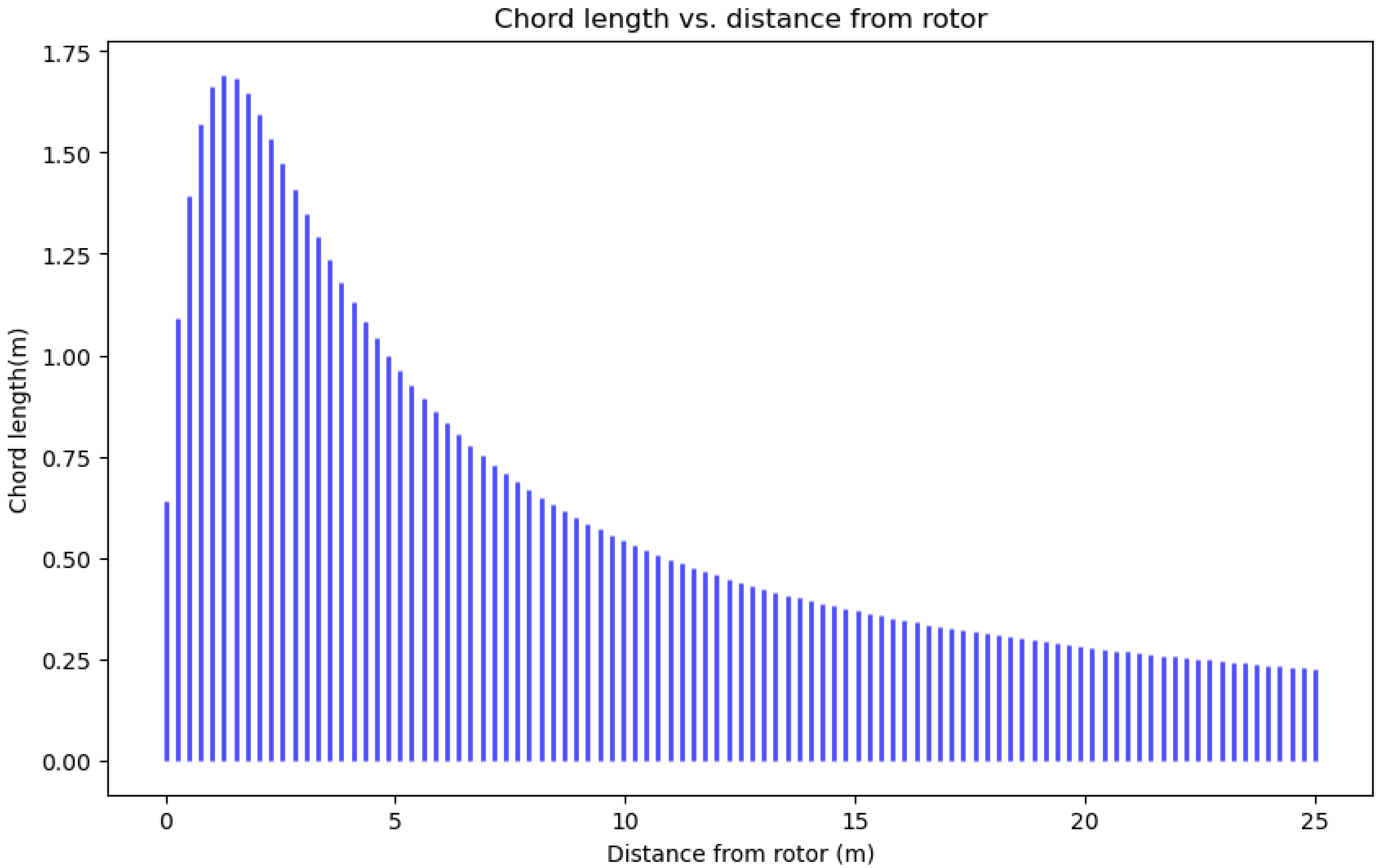
λ_r = Relative tip speed = $\lambda\left(\frac{r}{R}\right)$

R = Total blade length

In [4]:
```
def chordlen(r, R, CL):
    specific_tipspeed = tipspeed * r / R
    phi_opt = 5  # Optimal angle of attack (degrees)
    phi_opt_radians = np.radians(phi_opt)  # Convert to radians
    relative_wind = 2/3*np.arctan(2 / (3*specific_tipspeed))
    return (8 * np.pi * r) / (numblades * CL) * (1 - np.cos(relative_wind))
```

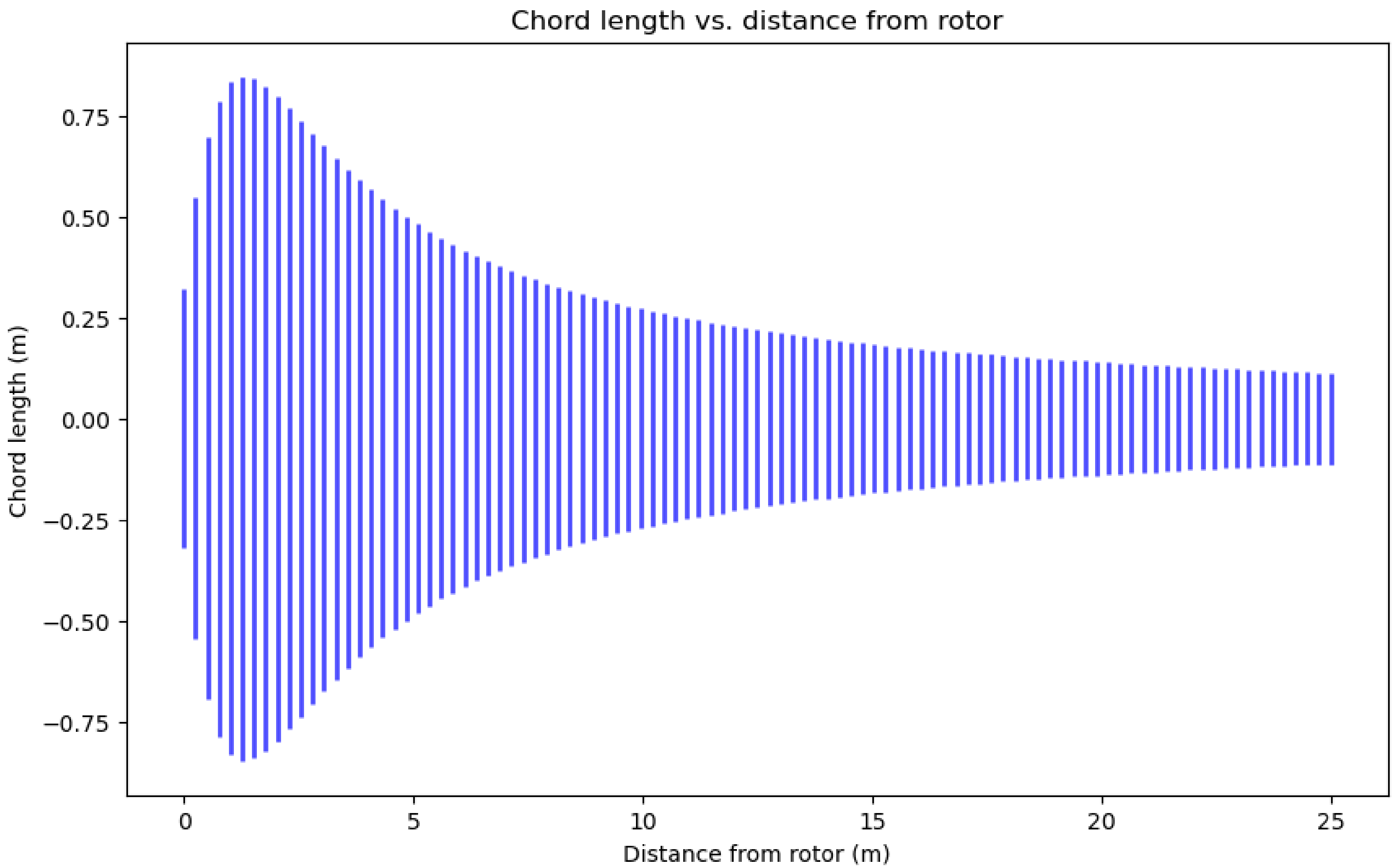
In [5]:
```
R = 25
tipspeed = 8
numblades = 3
chordlengths = []
CL = CLs[np.argmax(CLs/CDs)] #find the coefficient of lift at the optimal AoA

#creates a range of 0-25
r_range = np.linspace(0, R, 100)

#Iterates over each section of the blade and calculates chord length
for i in r_range[1:]:
    position = i
    chordlengths.append(chordlen(position, R, CL))

# for value in chordlengths:
#     print(value)
```

In [6]:
```
positions = np.linspace(0, R, len(chordlengths))


plt.figure(figsize=(10, 6))
# plt.xlim(4,25)
# plt.ylim(-25,25)
plt.title("Chord length vs. distance from rotor")
plt.xlabel("Distance from rotor (m)")
plt.ylabel("Chord length(m)")
for pos, chord in zip(positions, chordlengths):
    plt.vlines(x=pos, ymin=0, ymax=chord, color='blue', alpha=0.7, linewidth=2)

#plt.plot(positions, chordlengths)
```



In [7]:
```
positions = np.linspace(0, R, len(chordlengths))

plt.figure(figsize=(10, 6))
# plt.xlim(0, 12)
# plt.ylim(-25, 25)
plt.title("Chord length vs. distance from rotor")
plt.xlabel("Distance from rotor (m)")
plt.ylabel("Chord length (m)")
#plt.ylim(-25,25)
```

```
# Adjusting lines to center on the x-axis
for pos, chord in zip(positions, chordlengths):
    plt.vlines(x=pos, ymin=-chord/2, ymax=chord/2, color='blue', alpha=0.7, linewidth=2)

# Uncomment the following if you'd like to plot the data points as well
# plt.plot(positions, chordlengths, 'o', color='red')

plt.show()
```



Twist angle calculation

$$\theta = \phi - \alpha$$

θ = Twist angle

ϕ = Relative wind speed angle

α = Optimal angle of attack

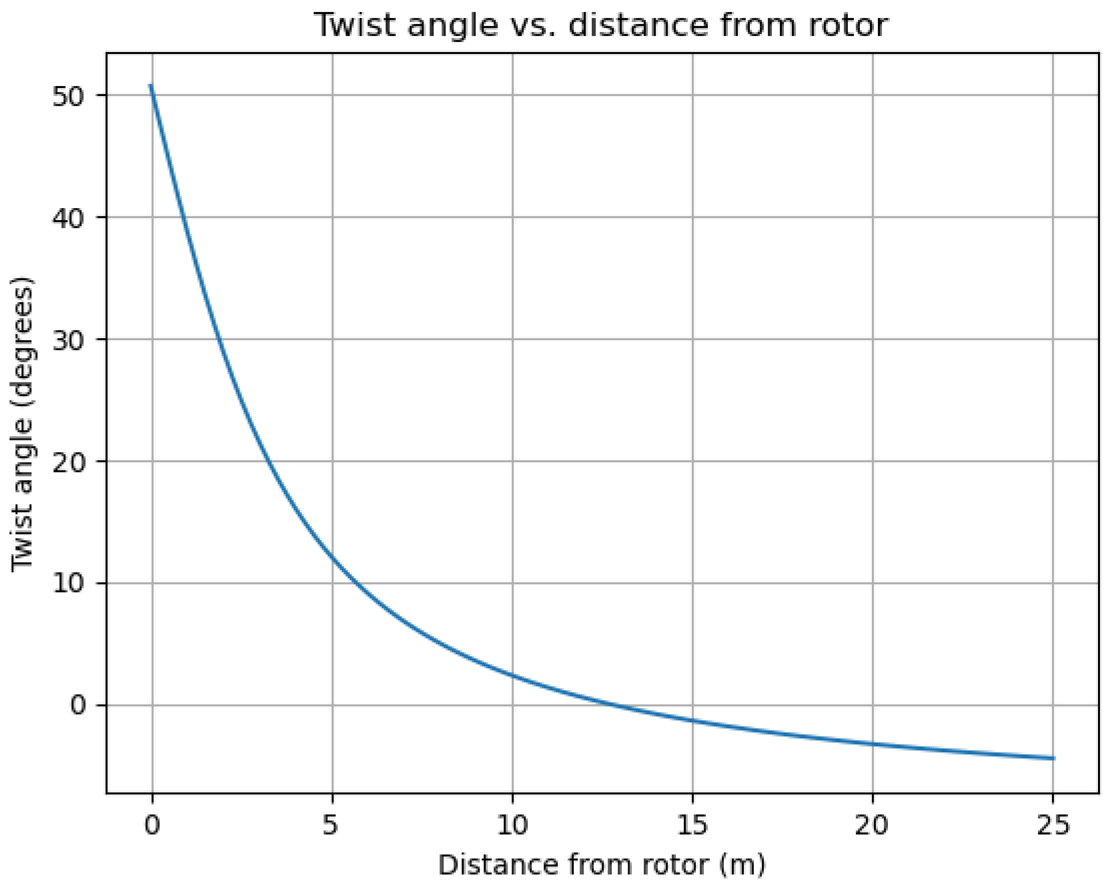
In [8]:

```
#Twist angle calculation
def twist_angle_func(r, R):
    specific_tipspeed = tipspeed * (r/R)
    relative_wind = 2/3*np.arctan(1 / specific_tipspeed)
    return relative_wind - np.radians(optaoa) # (rad)
```

In [9]:

```
#Make twist angle array
twist_angle = []
for r in r_range:
    twist_angle.append(twist_angle_func(r, R))
twist_angle_degrees = np.degrees(twist_angle)


# for i in twist_angle:
#print(i)


plt.plot(r_range,twist_angle_degrees)
plt.ylabel("Twist angle (degrees)")
plt.xlabel("Distance from rotor (m)")
plt.title("Twist angle vs. distance from rotor")
plt.grid()
plt.show()
```

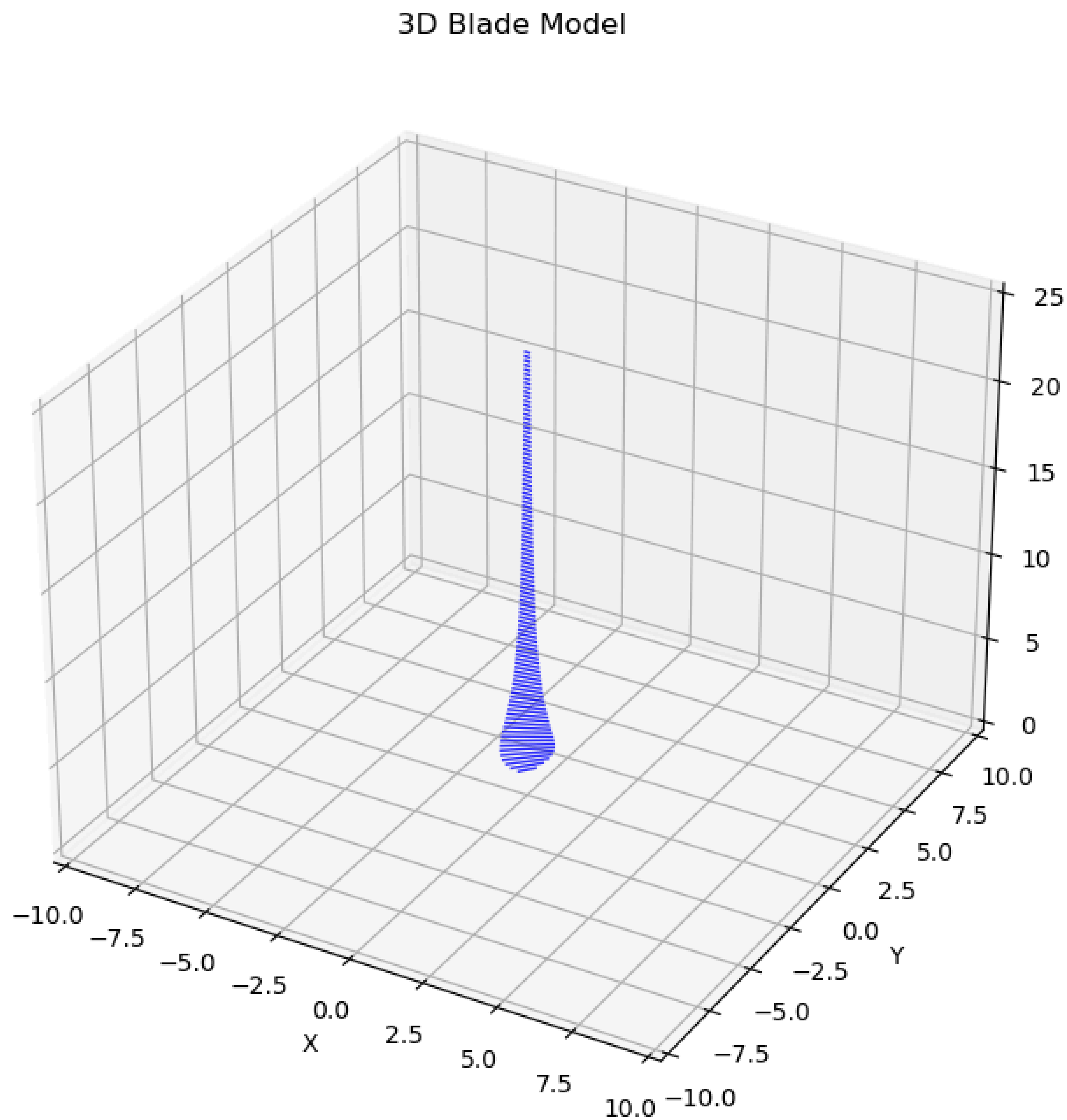
```
/var/folders/r9/70dwbmk11jv7m9tx2nkc7z_h0000gn/T/ipykernel_15268/2778075041.py:4: RuntimeWarning: divide by zero encoun
tered in scalar divide
  relative_wind = 2/3*np.arctan(1 / specific_tipspeed)
```



In [10]:
```
from mpl_toolkits.mplot3d.art3d import Line3DCollection


chordlengths = np.array(chordlengths) #makes chordlengths an np array for easier manipulation

# Blade sections - one line per meter
r = np.linspace(1, R, len(positions)-1)



# arrays for line segments
lines = []

for i, r_i in enumerate(r):
    c_i = chordlengths[i]
    twist_i = twist_angle[i]  # Twist angle in radians
```

```
    dx = (c_i / 2) * np.cos(twist_i)
    dy = (c_i / 2) * np.sin(twist_i)

    # Endpoint 1
    x1 = dx
    y1 = dy
    z1 = r_i

    # Endpoint 2
    x2 = -dx
    y2 = -dy
    z2 = r_i

    # Add the line segment connecting the two points
    lines.append([(x1, y1, z1), (x2, y2, z2)])

# Create a 3D plot
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

# Plot the line segments as a collection
line_collection = Line3DCollection(lines, linewidths=1, colors='blue', alpha=0.8)
ax.add_collection3d(line_collection)

#ax.view_init(90,0,0)  # Example: 30° elevation and 45° azimuth

# Set the axes labels
ax.set_title("3D Blade Model")
ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")

# Set the view limits to better visualize the blade
# ax.set_xlim(-max(chordlengths),max(chordlengths))
# ax.set_ylim(-max(chordlengths),max(chordlengths))
horizontallim = 10
ax.set_xlim(-horizontallim,horizontallim)
ax.set_ylim(-horizontallim,horizontallim)
ax.set_zlim(0, 25)
```

Out[10]:
```
(0.0, 25.0)
```


3D Blade Model
25
20
15
10
5
0
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
X

In [ ]: